Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

MIN-Fakultät
Fachbereich Informatik
Arbeitsbereich SAV/BV (KOGS)

Image Processing 1 (IP1)
Bildverarbeitung 1

Lecture 5 – Perspective Transformations and Interpolation

Winter Semester 2014/15

Slides: Prof. Bernd Neumann
Slightly revised by: Dr. Benjamin Seppke & Prof. Siegfried Stiehl

Perspective Projection Transformation

Where does a point of a scene appear in an image?

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{\;?\;} \begin{bmatrix} x_p'' \\ y_p'' \end{bmatrix}$$

Transformation in 3 steps:

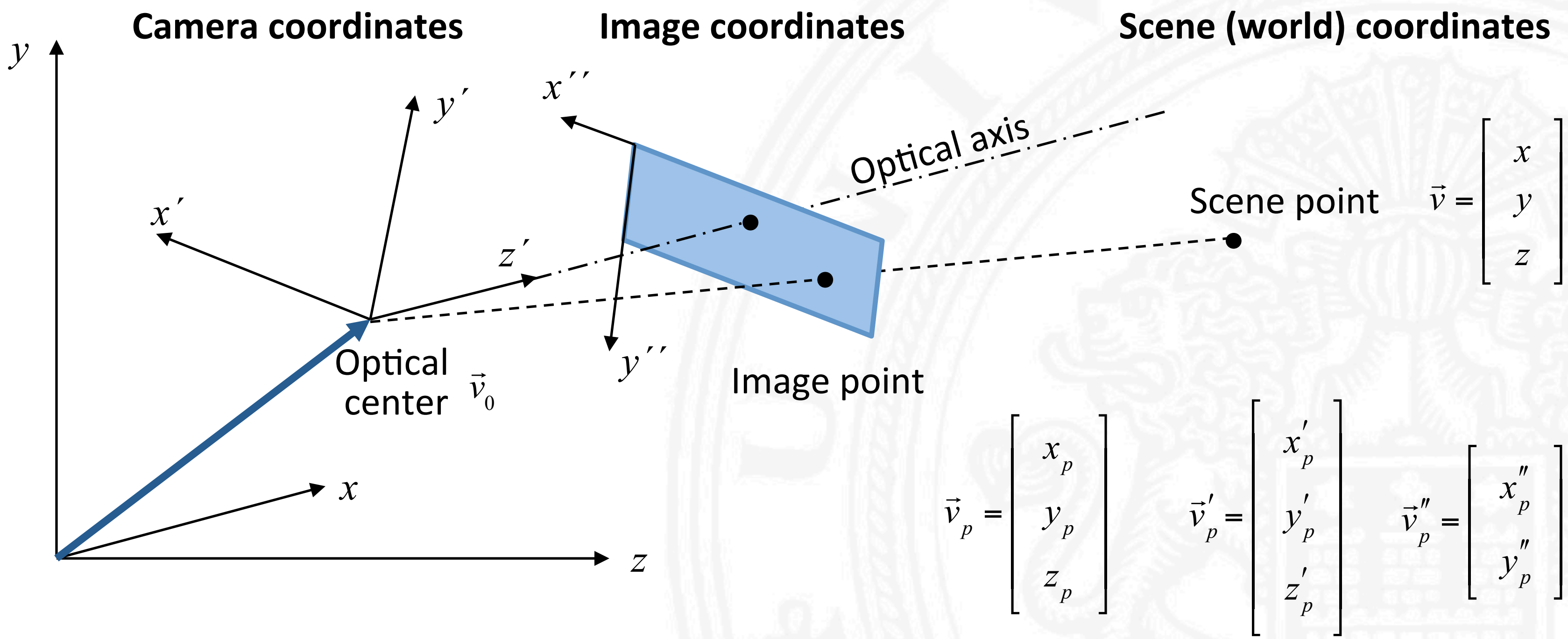
1. scene coordinates => camera coordinates
2. projection of camera coordinates into image plane
3. camera coordinates => image coordinates

Perspective projection equations are essential for Computer Graphics. For Image Understanding we will need the inverse: What are possible scene coordinates of a point visible in the image? This will follow later.

Perspective Projection in Independent Coordinate Systems

It is often useful to describe real-world points, camera geometry and image points in separate coordinate systems.

The formal description of projection involves transformations between these coordinate systems.



3D Coordinate Transformation I

The new coordinate system is specified by a <u>translation</u> and <u>rotation</u> with respect to the old coordinate system:

$$\vec{v}' = R\,(\vec{v} - \vec{v}_0)$$

$\vec{v}_0$ Optical center

R Rotation matrix

R may be decomposed into 3 rotations about the coordinate axes:
$R = R_x\, R_y\, R_z$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If rotations are performed in the above order:

1) γ = rotation angle about z-axis „tilt“
2) β = rotation angle about (new) y-axis „pan“
3) α = rotation angle about (new) x-axis „nick“

Note that these matrices describe coord. transforms for <u>positive</u> rotations of the coord. system.

3D Coordinate Transformation II

By multiplying the 3 matrices R_x, R_y and R_z one gets

$$R = \begin{bmatrix} \cos\beta\cos\gamma & \cos\beta\sin\gamma & -\sin\beta \\ \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\cos\beta \\ \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\cos\beta \end{bmatrix}$$

For formula manipulations, one tries to avoid the trigonometric functions and takes

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Note that the coefficients of R are constrained: A rotation matrix is <u>orthonormal</u>:

$R\,R^T = I$ **(unit matrix)**

Example for Coordinate Transformation

Camera coordinate system :

- Displacement by $\vec{v}_0$
- Rotation by pan angle $\beta = -30^0$
- Rotation by nick angle $\alpha = 45^0$

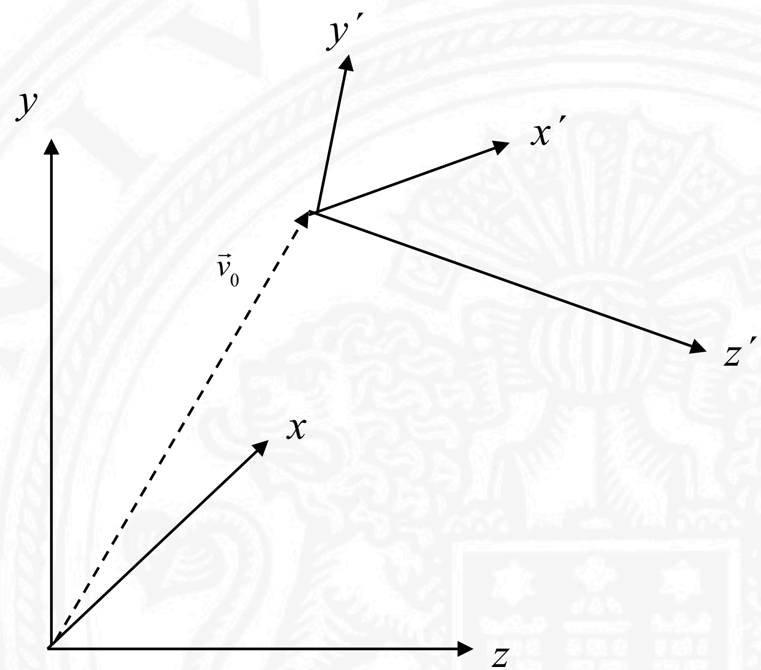
Application of:

$$\vec{v}' = R\ (\vec{v}' - \vec{v}_0) \ \text{ with } R = R_x R_y$$

and:

$$R_x = \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & \sqrt{2} & \sqrt{2} \\ 0 & -\sqrt{2} & \sqrt{2} \end{bmatrix}$$

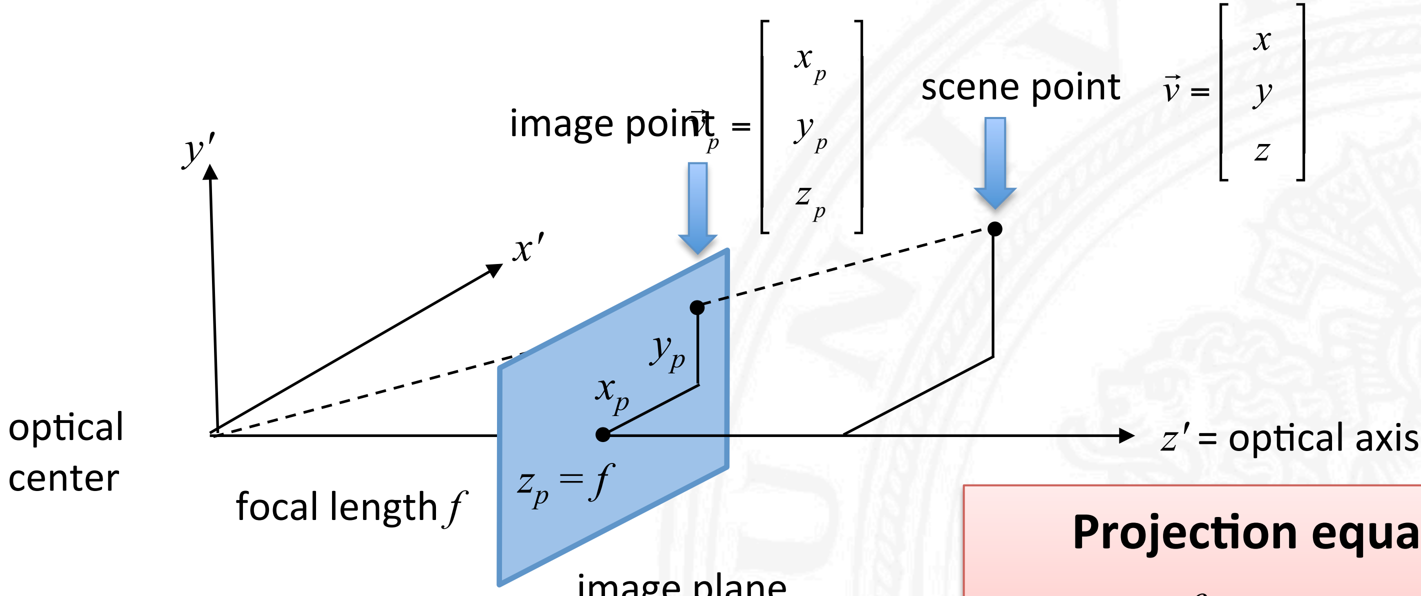
$$R_y = \frac{1}{2}\begin{bmatrix} \sqrt{3} & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & \sqrt{3} \end{bmatrix}$$



Perspective Projection Geometry

Projective geometry relates the coordinates of a point in a scene to the coordinates of its projection onto an image plane.

Perspective projection is an adequate model for most cameras.



Projection equations:

$$x_p = \frac{x\,f}{z} \qquad y_p = \frac{y\,f}{z}$$

Perspective and Orthographic Projection

Perspective Projektion:

- Projection equations

$$x_p = \frac{x\,f}{z} \quad y_p = \frac{y\,f}{z} \quad z_p = f$$ (f = focal length)

- Nonlinear transformation
- Loss of information

If all objects are far away (z‘ is large), f/z is approximately constant.
→ **Orthographic projection**:

$$x_p = s\,x \quad y_p = s\,y \quad z_p = f$$ (s = scaling factor)

– can be viewed as projection with parallel rays + scaling
– has some linear properties, commonly used for formal analysis.

From Camera Coordinates to Image Coordinates

Transform may be necessary because

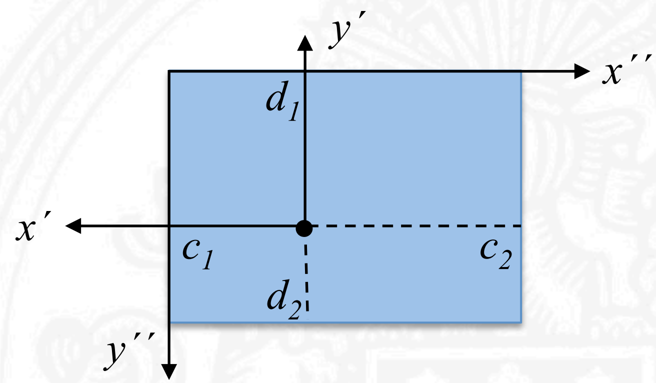
- optical axis may not penetrate image plane at origin of desired coordinate system
- transition to discrete coordinates may require scaling.

$$x''_p = \left(x'_p - x'_{p_0}\right) a \qquad a,\ b \qquad \text{scaling parameter}$$

$$y''_p = \left(y'_p - y'_{p_0}\right) b \qquad x'_{p_0},\ y'_{p_0} \qquad \text{origin of the image coordinate system}$$

Example:

- Image boundaries in camera coordinates:
  $x'_{max} = c_1 \quad x'_{min} = c_2$
  $y'_{max} = d_1 \quad y'_{min} = d_2$
- Discrete image coordinates:
  $x'' = 0 .. 511 \quad y'' = 0 .. 575$



Transformation parameters: $x'_{p_0} = c_1, \quad y'_{p_0} = d_1, \quad a = \frac{512}{c_2 - c_1}, \quad b = \frac{576}{d_2 - d_1}$

Complete Perspective Projection Equations

Combination of 3 transformation steps:

1. Scene coordinates → camera coordinates
2. Projection of camera coordinates into image plane
3. Camera coordinates → image coordinates

$$x_p'' = \left[\tfrac{f}{z'}\Big(\cos(\beta)\cos(\gamma)(x-x_0)+\cos(\beta)\sin(\gamma)(y-y_0)+\sin(\beta)(z-z_0)\Big)-x_{p_0}\right]a$$

$$y_p'' = \left[\frac{f}{z'}\begin{pmatrix} & (-\sin(\alpha)\sin(\beta)\cos(\gamma)-\cos(\alpha)\sin(\gamma))(x-x_0) \\ + & (-\sin(\alpha)\sin(\beta)\sin(\gamma)+\cos(\alpha)\cos(\gamma))(y-y_0) \\ + & \sin(\alpha)\cos(\beta)(z-z_0) \end{pmatrix} - y_{p_0}\right]b$$

with:

$$z' = \begin{pmatrix} & (-\cos(\alpha)\sin(\beta)\cos(\gamma)+\sin(\alpha)\sin(\gamma))(x-x_0) \\ + & (-\cos(\alpha)\sin(\beta)\sin(\gamma)-\sin(\alpha)\cos(\gamma))(y-y_0) \\ + & \cos(\alpha)\cos(\beta)(z-z_0) \end{pmatrix}$$

Homogeneous Coordinates I

4D notation for 3D coordinates which allows to express nonlinear 3D transformations as linear 4D transformations.

- Normal (3D): $\vec{v}' = R_{3x3}(\vec{v} - \vec{v}_0)$
- Homogeneous coordinates: $\vec{v}' = R_{4\times4} T_{4\times4} \vec{v} = A_{4\times4} \vec{v}$

$$R_{4\times4} T_{4\times4} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transition to homogeneous coordinates:

$\vec{v}^T = [x\ y\ z] \xrightarrow{affin} (\vec{v}_4)^T = [wx\ wy\ wz\ w]$ $w \neq 0$ is arbitrary constant

Return to normal coordinates:

(i) Divide components 1 to 3 by 4th component

(ii) Omit 4th component

Homogeneous Coordinates II

Perspective Projection in homogeous coordinates:

$$\vec{v}'_{p,4} = P_{4\times4}\vec{v}'_4 \quad \text{with } P_{4\times4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \quad \text{and } \vec{v}'_4 = \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix} \quad \text{gives: } \vec{v}_{p,4} = \begin{bmatrix} wx \\ wy \\ wz \\ \frac{wz}{f} \end{bmatrix}$$

Return to normal coordinates gives:

$$v'_p = \begin{bmatrix} \frac{xf}{z} \\ \frac{yf}{z} \\ f \end{bmatrix}$$

Compare with earlier slide!

Transformation from camera- to image coordinates:

$$\vec{v}''_{p,4} = B_{4\times4}\vec{v}'_{p,4} \quad \text{with } B_{4\times4} = \begin{bmatrix} a & 0 & 0 & -x_0 a \\ 0 & b & 0 & -y_0 b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and } \vec{v}'_{p,4} = \begin{bmatrix} wx_p \\ wy_p \\ 0 \\ w \end{bmatrix} \quad \text{gives: } \vec{v}''_{p,4} = \begin{bmatrix} wa(x_p - x_0) \\ wa(y_p - y_0) \\ 0 \\ w \end{bmatrix}$$

Homogeneous Coordinates III

Perspective projection can be completely described in terms of a linear transformation in homogeneous coordinates:

$$\vec{v}''_{p,4} = B_{4\times4} P_{4\times4} R_{4\times4} T_{4\times4} \vec{v}_4$$

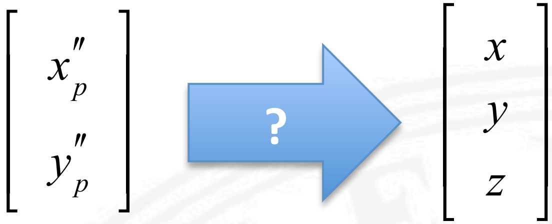
$B_{4\times4} P_{4\times4} R_{4\times4} T_{4\times4}$ may be combined into a single 4x4-Matrix C :

$$\vec{v}''_{p,4} = C_{4\times4} \vec{v}_4$$

In the literature the parameters of these equations may vary because of different choices of coordinate systems, different order of translation and rotation, different camera models, etc.

Inverse Perspective Equations

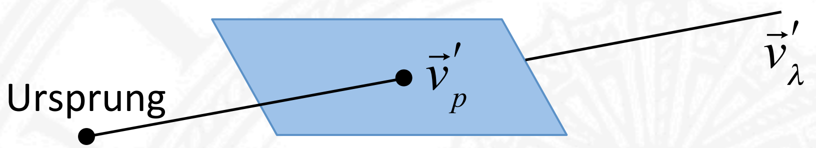
Which points in a scene correspond to a point in the image?



Each image point defines a projection ray as the locus of possible scene points (for simplicity in camera coordinates):

$\vec{v}'_p \rightarrow \vec{v}'_\lambda = \lambda \vec{v}'_p$ (λ: free parameter)

$$\boxed{\vec{v} = \vec{v}_0 + R^T \lambda \vec{v}'_p}$$

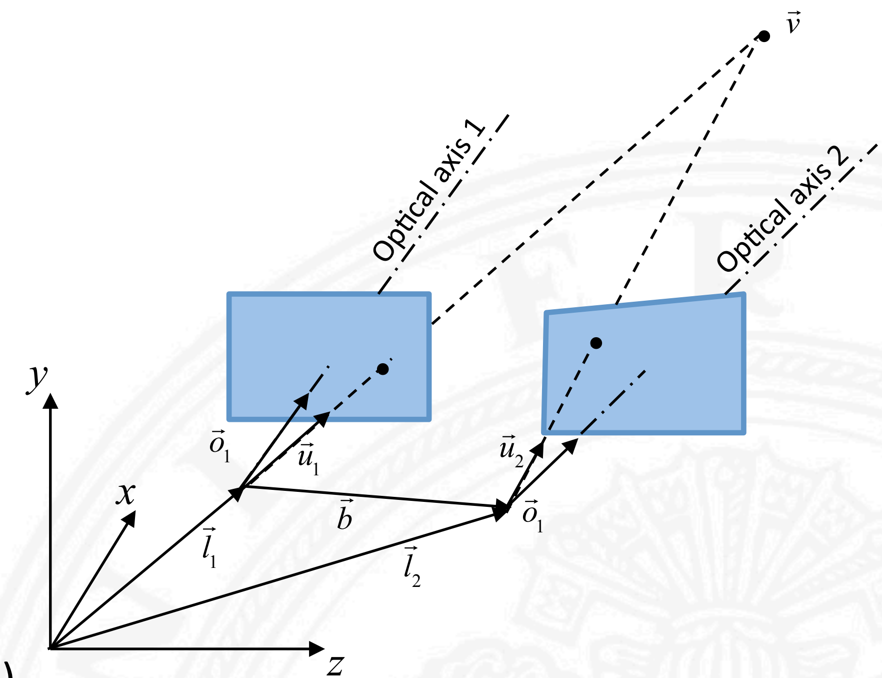


Result: 3 equations with the 4 unknowns x, y, z, λ and camera parameters R and $\vec{v}_0$

Applications of inverse perspective mapping for e.g.

- distance measurements
- binocular stereo, motion stereo,
- camera calibration

Binocular Stereo I



$\vec{l}_1, \vec{l}_2$ Camera positions (optical centers)

$\vec{b}$ Stereo base (baseline)

$\vec{o}_1, \vec{o}_2$ Camera orientations (unit vectors)

$\vec{f}_1, \vec{f}_2$ Focal lengths

$\vec{v}$ Scene points

$\vec{u}_1, \vec{u}_2$ Projection rays of scene point (unit vectors)

Binocular Stereo II

Determine the distance to $\vec{v}$ by measuring $\vec{u}_1$ and $\vec{u}_2$.

Formally: $\alpha\vec{u}_1 = \vec{b} + \beta\vec{u}_2 \quad \Rightarrow \quad \vec{v} = \alpha\vec{u}_1 + \vec{l}_1$

α and β are overconstrained by the vector equation. In practice, measurements are inexact, no exact solution exists (rays do not intersect).

Better approach: Solve for the point of closest approximation of both rays:

$$\vec{v} = \frac{\alpha_0\vec{u}_1 + \left(\vec{b} + \beta_0\vec{u}_2\right)}{2} + \vec{l}_1 \quad \Rightarrow \quad \textbf{minimize: } \left\|\alpha_0\vec{u}_1 + \left(\vec{b} + \beta_0\vec{u}_2\right)\right\|^2$$

Minimization: $\left\|\alpha_0\vec{u}_1 + \left(\vec{b} + \beta_0\vec{u}_2\right)\right\|^2 = \left(\alpha_0\vec{u}_1 + \left(\vec{b} + \beta_0\vec{u}_2\right)\right)^{\mathrm{T}}\left(\alpha_0\vec{u}_1 + \left(\vec{b} + \beta_0\vec{u}_2\right)\right)$

$$\begin{aligned}
\frac{\partial}{\partial\alpha_0}\left(\alpha_0\vec{u}_1 + \left(\vec{b} + \beta_0\vec{u}_2\right)\right)^{\mathrm{T}}\left(\alpha_0\vec{u}_1 + \left(\vec{b} + \beta_0\vec{u}_2\right)\right) &= \vec{u}_1^{\mathrm{T}}\left(\alpha_0\vec{u}_1 - \left(\vec{b} + \beta_0\vec{u}_2\right)\right) + \left(\alpha_0\vec{u}_1 - \left(\vec{b} + \beta_0\vec{u}_2\right)\right)^{\mathrm{T}}\vec{u}_1 \\
&= 2\,\vec{u}_1^{\mathrm{T}}\left(\alpha_0\vec{u}_1 - \left(\vec{b} + \beta_0\vec{u}_2\right)\right) \\
&= 2\left(\alpha_0 - \vec{u}_1^{\mathrm{T}}\vec{b} - \beta_0\vec{u}_1^{\mathrm{T}}\vec{u}_2\right) = 0
\end{aligned}$$

Binocular Stereo III

$$2\left(\alpha_0 - \vec{u}_1^{\mathrm{T}}\vec{b} - \beta_0\vec{u}_1^{\mathrm{T}}\vec{u}_2\right) = 0 \quad\Rightarrow\quad \alpha_0 = \vec{u}_1^{\mathrm{T}}\vec{b} + \beta_0\vec{u}_1^{\mathrm{T}}\vec{u}_2 \qquad (1)$$

$$\frac{\partial}{\partial\beta_0}\left(\alpha_0\vec{u}_1 + \left(\vec{b}+\beta_0\vec{u}_2\right)\right)^{\mathrm{T}}\left(\alpha_0\vec{u}_1 + \left(\vec{b}+\beta_0\vec{u}_2\right)\right) = -\vec{u}_2^{\mathrm{T}}\left(\alpha_0\vec{u}_1 - \left(\vec{b}+\beta_0\vec{u}_2\right)\right) - \left(\alpha_0\vec{u}_1 - \left(\vec{b}+\beta_0\vec{u}_2\right)\right)^{\mathrm{T}}\vec{u}_2$$

$$= -2\,\vec{u}_2^{\mathrm{T}}\left(\alpha_0\vec{u}_1 - \left(\vec{b}+\beta_0\vec{u}_2\right)\right)$$

$$= -2\left(\alpha_0\vec{u}_2^{\mathrm{T}}\vec{u}_1 - \vec{u}_2^{\mathrm{T}}\vec{b} - \beta_0\right)$$

$$-2\left(\alpha_0\vec{u}_2^{\mathrm{T}}\vec{u}_1 - \vec{u}_2^{\mathrm{T}}\vec{b} - \beta_0\right) = 0 \quad\Rightarrow\quad \beta_0 = -\vec{u}_1^{\mathrm{T}}\vec{b} + \alpha_0\vec{u}_2^{\mathrm{T}}\vec{u}_1 \qquad (2)$$

Insert (2) into (1) gives:

$$\alpha_0 = \frac{\vec{u}_1^T\vec{b} - \left(\vec{u}_1^T\vec{u}_2\right)\left(\vec{u}_2^T\vec{b}\right)}{1-\left(\vec{u}_1^T\vec{u}_2\right)} \qquad \beta_0 = \frac{\left(\vec{u}_1^T\vec{u}_2\right)\left(\vec{u}_1^T\vec{b}\right) - \left(\vec{u}_2^T\vec{b}\right)}{1-\left(\vec{u}_1^T\vec{u}_2\right)^2}$$

Distance in Digital Images

Intuitive concepts of continuous images do not always carry over to digital images.

Several methods for measuring distance between pixels:

Eucledian distance

$$D_E((i,j),(h,k)) = \sqrt{(i-h)^2 + (j-k)^2}$$

costly computation of square root,
can be avoided for distance comparisons

City-block distance

$$D_4((i,j),(h,k)) = |i-h| + |j-k|$$

number of horizontal and vertical steps in a rectangular grid

Chessboard distance

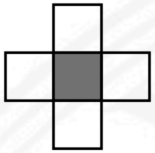
$$D_8((i,j),(h,k)) = \max\{|i-h|, |j-k|\}$$

number of steps in a rectangular grid if diagonal steps are allowed (number of moves of a king on a chessboard)

Connectivity in Digital Images

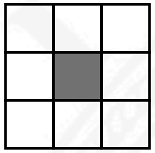
Connectivity is an important property of subsets of pixels. It is based on adjacency (or neighbourhood):

Pixels are 4-neighbours if their distance is $D_4 = 1$



all 4-neighbours of center pixel

Pixels are 8-neighbours if their distance is $D_8 = 1$



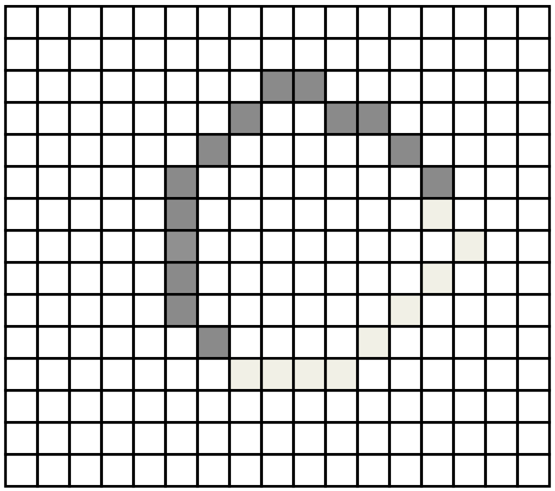
all 8-neighbours of center pixel

A path from pixel P to pixel Q is a sequence of pixels beginning at Q and ending at P, where consecutive pixels are neighbours.

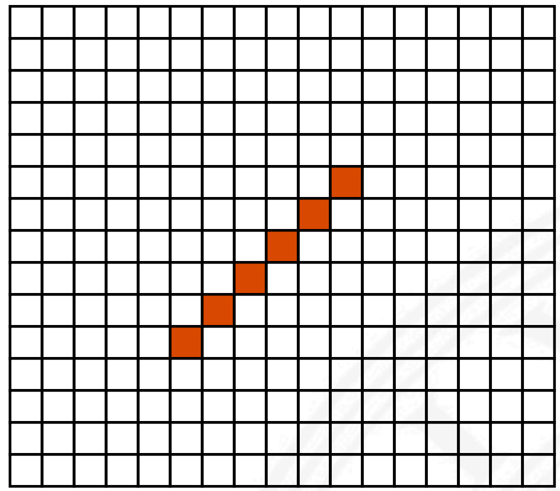
In a set of pixels, two pixels P and Q are connected, if there is a path between P and Q with pixels belonging to the set.

A region is a set of pixels where each pair of pixels is connected.

Closed Curve Paradoxon

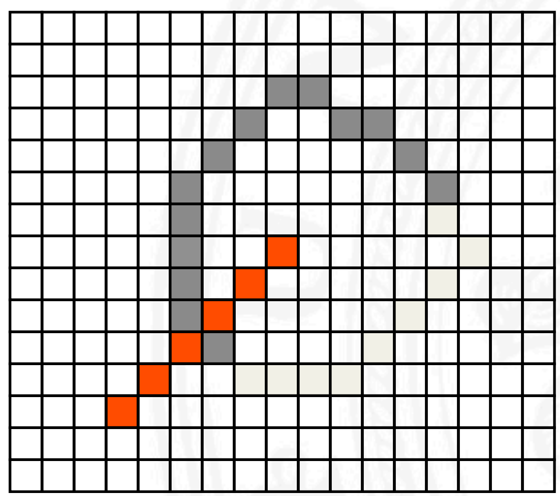


line 1



line 2

Solid lines if 8-pixel neighbourhood is used!



Line 2 does not intersect line 1 although it crosses from the outside to the inside!

A similar paradoxon arises if 4-pixel neighbourhoods are used!

Geometric Transformations

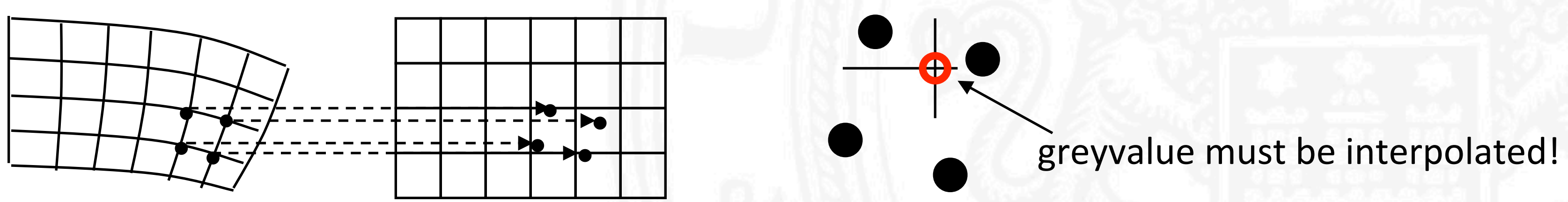
Various applications:

- change of view point
- elimination of geometric distortions from image capturing
- registration of corresponding images
- artificial distortions, Computer Graphics applications

Step 1: **Determine mapping $\underline{T}(x, y)$ from old to new coordinate system**

Step 2: **Compute new coordinates (x', y') for (x, y)**

Step 3: **Interpolate greyvalues at grid positions from greyvalues at transformed positions**



Polynomial Coordinate Transformations

General format of transformation:

$$x' = \sum_{i=0}^{m} \sum_{k=0}^{m-i} a_{ik} x^i y^k \qquad y' = \sum_{i=0}^{m} \sum_{k=0}^{m-i} b_{ik} x^i y^k$$

Assume polynomial mapping between (x, y) and (x', y') of degree m

Determine corresponding points

a) Solve linear equations for a_{ik}, b_{ik} ($i, k = 1 \ldots m$)

b) Minimize mean square error (MSE) for point correspondences

Approximation by biquadratic transformation:

$$x' = a_{00} + a_{10}x + a_{01}y + a_{11}xy + a_{20}x^2 + a_{02}y^2$$

$$y' = b_{00} + b_{10}x + b_{01}y + b_{11}xy + b_{20}x^2 + b_{02}y^2$$

at least 6 corresponding pairs needed

Approximation by affine transformation:

$$x' = a_{00} + a_{10}x + a_{01}y$$

$$y' = b_{00} + b_{10}x + b_{01}y$$

at least 3 corresponding pairs needed

Translation, Rotation, Scaling, Skewing

- **Translation** by vector $\vec{t}$:

$$\vec{v}\,' = \vec{v} + \vec{t} \quad \text{with} \quad \vec{v}\,' = \begin{bmatrix} x' \\ y' \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad \vec{\mathrm{t}} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

- **Rotation** of image coordinates by angle α:

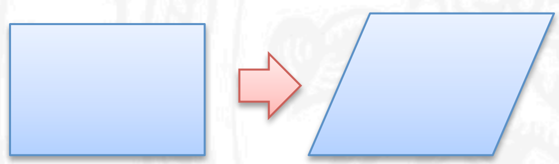
$$\vec{v}\,' = R\ \vec{v} \quad \text{with} \quad \mathrm{R} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

- **Scaling** by factor a in x-direction and factor b in y-direction:

$$\vec{v}\,' = S\ \vec{v} \quad \text{with} \quad \mathrm{S} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

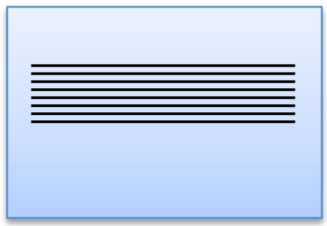
- **Skewing** by angle β:

$$\vec{v}\,' = W\ \vec{v} \quad \text{with} \quad W = \begin{bmatrix} 1 & \tan\beta \\ 0 & 1 \end{bmatrix}$$

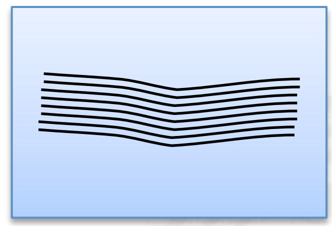


Example of Geometry Correction by Scaling

Distortions of electron-tube cameras may be 1 - 2 % => more than 5 lines for TV images



ideal image



actual image

Correction procedure may be based on

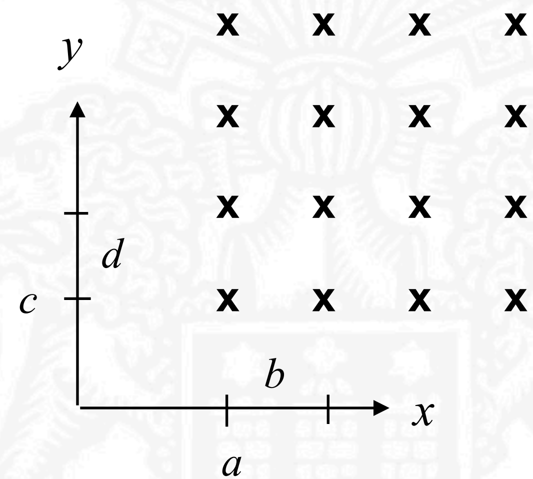
- fiducial marks engraved into optical system
- a test image with regularly spaced marks

Ideal mark positions:

$x_{mn} = a + mb \; , \; y_{mn} = c + nd \quad \text{with} \;\; m = 0 \ldots M-1 \text{ and n} = 0 \ldots N-1$

Actual mark positions:

$x'_{mn}, \; y'_{mn}$



Determine a, b, c, d **such that MSE (mean square error) of deviations is minimized**

Minimizing the MSE

Minimize
$$E = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}(\mathrm{x}_{\mathrm{mn}} - x'_{mn})^2 + (\mathrm{y}_{\mathrm{mn}} - y'_{mn})^2$$
$$= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}(\mathrm{a{+}mb} - x'_{mn})^2 + (\mathrm{c} + nd - y'_{mn})^2$$

From $\frac{\partial E}{\partial a} = \frac{\partial E}{\partial b} = \frac{\partial E}{\partial c} = \frac{\partial E}{\partial d} = 0$ we get:

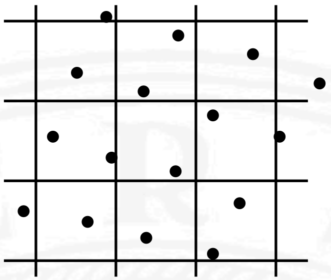
$$a = \frac{2}{MN(M+1)}\sum_m\sum_n(2M-1-3m)x'_{mn}$$
$$b = \frac{6}{MN(M^2-1)}\sum_m\sum_n(2m-M+1)x'_{mn}$$
$$c = \frac{2}{MN(N+1)}\sum_m\sum_n(2N-1-3n)y'_{mn}$$
$$d = \frac{6}{MN(N^2-1)}\sum_m\sum_n(2n-N+1)y'_{mn}$$

Special case $M{=}N{=}2$**:**

$$a = \tfrac{1}{2}(x'_{00} + x'_{01})$$
$$b = \tfrac{1}{2}(x'_{10} - x'_{00} + x'_{11} - x'_{01})$$
$$c = \tfrac{1}{2}(y'_{00} + y'_{01})$$
$$d = \tfrac{1}{2}(y'_{10} - y'_{00} + y'_{11} - y'_{01})$$

Principle of Greyvalue Interpolation

Greyvalue interpolation = computation of unknown greyvalues at locations (u´v´) from known greyvalues at locations (x´y´)



Two ways of viewing interpolation in the context of geometric transformations:

A) Greyvalues at grid locations (x y) in old image are placed at corresponding locations (x´y´) in new image: g(x´y´) = g(T(x y))

→ interpolation in new image

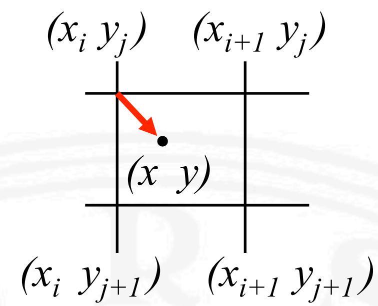
B) Grid locations (u´v´) in new image are transformed into corresponding locations (u v) in old image: g(u v) = g(T^{-1}(u´v´))

→ interpolation in old image

We will take view B:

Compute greyvalues between grid from greyvalues at grid locations.

Nearest Neighbour Greyvalue Interpolation

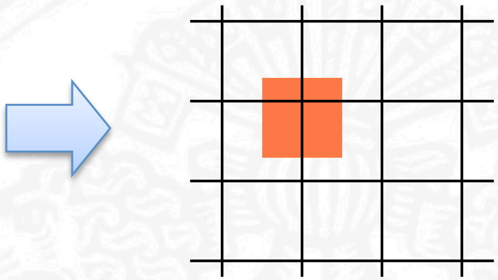


Assign $(x\ y)$ to greyvalue of nearest grid location

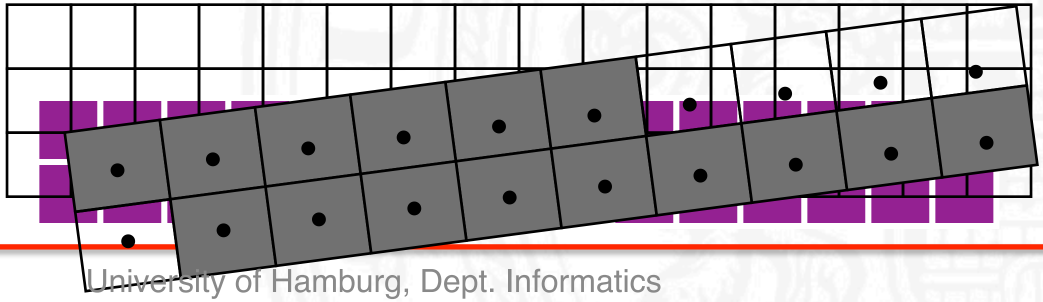
$(x_i\ \ y_j)\ (x_{i+1}\ \ y_j)\ (x_i\ \ y_{j+1})\ (x_{i+1}\ \ y_{j+1})$ grid locations

$(x\ \ y)$ location between grid
with $x_i \leq x \leq x_{i+1}$
and $y_j \leq y \leq y_{j+1}$

Each grid location represents the greyvalues in a rectangle centered around this location:



Straight lines or edges may appear step-like after this transformation:



Bilinear Greyvalue Interpolation

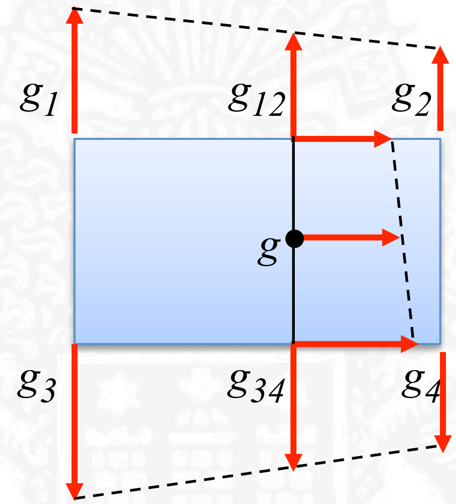
The greyvalue at location $(x\ y)$ between 4 grid points $(x_i\ y_j)$ $(x_{i+1}\ \ y_j)$ $(x_i\ \ y_{j+1})$ $(x_{i+1}\ \ y_{j+1})$ is computed by linear interpolation in both directions:

$$g(x,y) = \frac{1}{(x_{i+1}-x_i)(y_{j+1}-y_i)}\Big\{(x_{i+1}-x)(y_{j+1}-y)g(x_i,y_j) + (x-x_i)(y_{j+1}-y)g(x_{i+1},y_j) +$$

$$(x_{i+1}-x)(y-y_j)g(x_i,y_{j+1}) + (x-x_i)(y-y_j)g(x_{i+1},y_{j+1})\Big\}$$

Simple idea behind long formula:

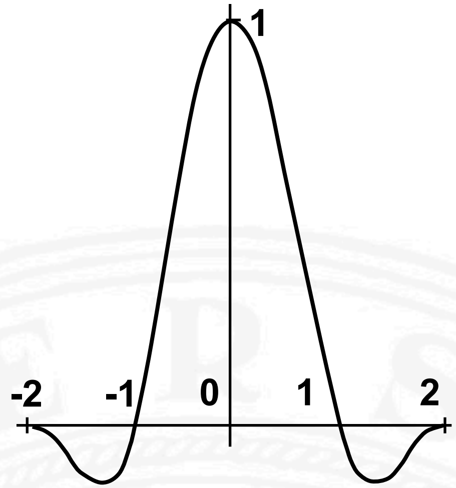
1. Compute g_{12} = linear interpolation of g_1 and g_2
2. Compute g_{34} = linear interpolation of g_3 and g_4
3. Compute g = linear interpolation of g_{12} and g_{34}



The step-like boundary effect is reduced.

But bilear interpolation may blur sharp edges.

Bicubic Interpolation



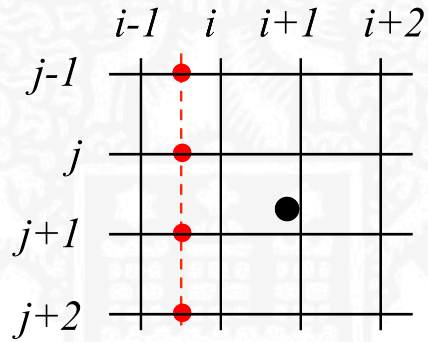
cross section of interpolation kernel

Each greyvalue at a grid point is taken to represent the center value of a local bicubic interpolation surface with cross section h_3.

$$h_3 = \begin{cases} 1 - 2|x|^2 + |x|^3 & \text{for } 0 < |x| \le 1 \\ 4 - 8|x| + 5|x|^2 - |x|^3 & \text{for } 1 < |x| < 2 \\ 0 & \text{else} \end{cases}$$

The greyvalue at an arbitrary point *(u v)* (black dot in figure) can be computed by

- four horizontal interpolations to obtain greyvalues at points *(u j-1)* ... *(u j+2)* (red dots), followed by
- one vertical interpolation (between red dots) to obtain greyvalue at *(u v)*.



<u>Note</u>: For an image with constant greyvalues g_0 the interpolated greyvalues at all points between the grid lines are also g_0.